FINAL REPORT

"ENGINEERING FRACTURE MECHANICS"

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Research performed wholly or partially with support of this contract, in various categories, will be summarized briefly. Numbers in brackets refer to the list of publications appended.

(a) *Tearing resistance in sheets and shells*

On the basis of the Dugdale model, approximate weight-function procedures for crack analysis, and a crack-opening shape criterion for continuing crack growth, theoretical curves for tearing resistance (J vs. crack growth Δc) were derived for thin flat sheets of finite size [1]. The substantial effects of finite specimen size and initial tearing resistance were thereby revealed.

This work was extended to circumferential cracks in cylindrical shells subjected to bending for which crack growth resistance curves (moment vs. Δc, J vs. Δc) were calculated via approximate analytical solutions of the semi-membrane equations of shell theory [9, 10].

(b) *Growing-crack asymptotics*

The singular fields around the tips of growing cracks in linear-hardening materials were calculated for a variety of conditions (plane stress, plane strain, modes I, II, III). [4, 5, 6]. The actual magnitude of the singularity strength (in addition to its type) was also calculated, for steady crack growth, as a function of the applied stress-intensity factor and the strain-hardening of the material [7].

(c) ***Toughened ceramics***

(i) *Ductile particle toughening*. The increase in fracture toughness of brittle ceramics via small, ductile-particle inclusions w ınalyzed on the basis of their effects in bridging the opposıng faces of a crack as it grows in the matrix [11]. The effects of particle size, ductility, and strength were elucidated, resistance curves were d :rived, and correlations were made between toughening and bridging length. This initial study was based on the assumption of small-scale bridging.

(ii) *Fiber toughening*. The toughening effects of aligned, frictionally constrained fibers were analyzed [14]. Here too the primary toughening mechanisms is crack bridging. A correlation was made with the results of earlier work on the matrix cracking (with unbroken fibers) of fiber-reinforced ceramics, and the toughening was related directly to the critical matrix cracking stress and the fiber strength.

(iii) *Transformation toughening*. The toughening effects of phase-transforming particles (ZrO_2) were studied in a series of papers. In all of this work, a simple hydrostatic-stress criterion was assumed for the "supercritical" (i.e. complete) production of the phase transformation, which was assumed to be purely dilatational. A full solution for steady-state crack growth was carried out in [12], extending incomplete earlier work. The problem of transient crack growth in the presence of transforming particles was solved in [15] over the full range of growth from initiation.to steady-state. Remarkably, peak toughening was found to occur at finite amounts of crack growth.

Transformation *strengthening* may be defined as the increase in ultimate tensile strength, due to the presence of phase-transforming particles, when the strength is governed by the worst flaw in the brittle matrix. A study of transformation strengthening was carried out in [16] by analyzing the growth of cracks of finite size. Roughly speaking, it was found that "poor" material -- that is, material containing large flaws -- could be strengthened appreciably by phase-transforming particles; but material that is "good" to begin with can not be expected to gain much strength form ZrO_2 reinforcement.

(iv) *Synergistic effects.* The combined toughening effect of both ductile and transforming particles during steady-state crack growth were derived in [13]. Parametric ranges were discovered in which the separate effects combined synergistically, that is, the net toughening ratio was nearly the product of the individual toughening ratios. This work was extended in [17], wherein the transient crack-growth problem was solved, and similar synergistic ranges were discovered for peak toughening at finite amounts of crack growth.

(d) ***Cellular materials***

Three papers have been written on the elastic behavior of pressurized cellular materials [8, 18, 19], in connection with the properties of lungs. In this work a dodecahedral model was exploited as the representation of an equivalent cell in order to deduce increment elastic moduli in the presence of internal pressure.

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List of publications (including original Harvard Report number)

1. "On Size Effects in Plane Stress Crack-Growth Resistance", Bernard Budiansky and Eric E. Sumner, Jr., in **Developments in Mechanics,** Vol. 13, Proceedings of the 19th Midwestern Mechanics Conference, Ohio State University, Columbus, Ohio, September 9-11, 1985. MECH-68 (April 1985)]

2. "Micromechanics II" - Bernard Budiansky, in **Proceedings of 10th U. S. National Congress of Applied Mechanics**, Austin, Texas, June 16-20, 1986. [MECH-82 (June 1986)]

3. "Matrix Fracture in Fiber-Reinforced Ceramics" - B. Budiansky, J. W. Hutchinson and A. G. Evans, *Journal Mechanics and Physics of Solids*, Vol. 34, No. 2, 1986. [MECH-64 (March 1985)]

4. "Asymptotic Fields of a Perfectly Plastic, Plane Stress Mode II Growing Crack" - P. Ponte Castañeda, *Journal of Applied Mechanics*, Vol. 53, December 1986. [MECH-71 (August 1985)]

5. "Asymptotic Fields in Steady Crack Growth with Linear Strain-Hardening" - P. Ponte Castañeda, *Journal Mechanics and Physics of Solids*, Vol. 35, No. 2, 1987. [MECH-69 (August 1985)]

6. "Asymptotic Analysis of a Mode I Crack Propagating Steadily in a Deformation Theory Material" - P. Ponte Castañeda, *Journal of Applied Mechanics*, Vol. 54, March 1987. [MECH-70 (August 1985)]

7. "Plastic Stress Intensity Factors in Steady Crack Growth" - P. Ponte Castañeda, *Journal of Applied Mechanics*, Vol. 54, June 1987. [MECH-81 (June 1986)]

8. "Elastic Moduli of Lungs" - B. Budiansky and E. Kimmel, *Journal of Applied Mechanics*, Vol. 54, June 1987. [MECH-78 (May 1986)]

9. "Dugdale Model for Circumferential Through-Cracks in Pipes Loaded by Bending" - J. Lyell Sanders, Jr., *International Journal of Fracture*, Vol. 34, 1987. [MECH-77 (March 1986)]

10. "Tearing of Circumferential Cracks in Pipes Loading by Bending" - J. Lyell Sanders, Jr.. *International Journal of Fracture*, Vol. 35, 1987. [MECH-79 (May 1986)]

11. "Small-Scale Crack Bridging and the Fracture Toughness of Particulate-Reinforced Ceramics" - B. Budiansky, J. C. Amazigo and A. G. Evans, *Journal Mechanics and Physics of Solids*, Vol. 36, No. 2, 1988. [MECH-104 (June 1987)]

12. "Steady-State Crack Growth in Supercritically Transforming Materials" - J. C. Amazigo and B. Budiansky, *Int. J. Solids and Structures*, Vol. 24, 1988. [MECH-107 (August 1987)]

13. "Interaction of Particulate and Transformation Toughening" - J. C. Amazigo and B. Budiansky, *J. Mech. Phys. Solids*, Vol. 36, 1988. [MECH-112 (December 1987)]

14. "Toughening by Aligned, Frictionally Constrained Fibers" - B. Budiansky and J. C. Amazigo, *J. Mech. Phys. Solids*, Vol. 37, 1989. [MECH-119 (March 1988)]

15. "Crack-Growth Resistance in Transformation-Toughened Ceramics" - D. M. Stump and B. Budiansky, *Int. J. Solids and Structures*, Vol. 25, 1989. [MECH-124 (June 1988)]

16. "Finite Cracks in Transformation-Toughened Ceramics" - D. M. Stump and B. Budiansky, *Acta metall.*, Vol. 37, No. 12, 1989. [MECH-138 (February 1989)]

17. MECH-147 - "Crack Growth Resistance in Transformation-Toughened and Ductile-Particle Reinforced Ceramic" - David M Stump (July 1989). Submitted for publication in *International Journal of Solids and Structures*

18. MECH-149 - "Surface Tension and the Dodecahedron Model for Lung Elasticity" - Eitan Kimmel and Bernard Budiansky (September 1989). To be published in ASME *Journal of Biomechanical Engineering.*

19. MECH-150 - "On the Shear Modulus of Polyhedron-Cell Liquid Foam" - Bernard Budiansky and Eitan Kimmel (September 1989). To be published in ASME *Journal of Applied Mechanics.*